

QUANTIFICATION OF FLOW RESISTANCE FOR UNLINED CANALS IN ALLUVIAL SOIL

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Abstract: This paper reports a model developed to predict and quantify the flow resistance in alluvial unlined channels/canals from fresh data (2011) for Jamrao Canal (Sindh, Pakistan), collected in Ph.D. thesis. This data has been used to quantify the flow resistance coefficient 'n' in alluvial unlined channels/canals using dimensional analysis method, specific equations have been developed. Methods used for analysis are fitted line and matrix formation. Fitted line method proved to be convenient mathematical tool, while alternative tool of matrix formation was found unsuitable since it produce very low values. For comparison the values of 'n' have also been calculated for the same data by Lacey's equation.

Keywords: Alluvial channel, dimensional analysis, fitted lined method, flow resistance Lacey's equations, matrix formation method.

1. Introduction

The design theories and procedures of unlined alluvial channels have remained a subject of investigation for almost last 150 years. Pakistan's irrigation system is the world's third largest irrigation system [1, 2] (Figure 1) which consists of 63,100 km (39,209 miles) of unlined alluvial canals [3] (Table 1). Most of these canals had been designed as per Lacey's [4] theory that presented a concept of final regime. Based on this concept, developed a series of equations derived from field data of measurements in such canals which according to author were in final regime.

According to Lacey [4], the regime condition i.e. stable conditions vis-à-vis bed width, depth and slope, that is zero net erosion, or deposition over a hydrological cycle, shall be established when:

(1) discharge is constant; (2) the alluvium in which the channel is flowing is incoherent, unlimited and of the same characteristics as the sediment charge carried by the water. Incoherent alluvium means the loose granular material which can be scoured out as

easily as it is deposited; and (3) silt grade and charge are constant [1].

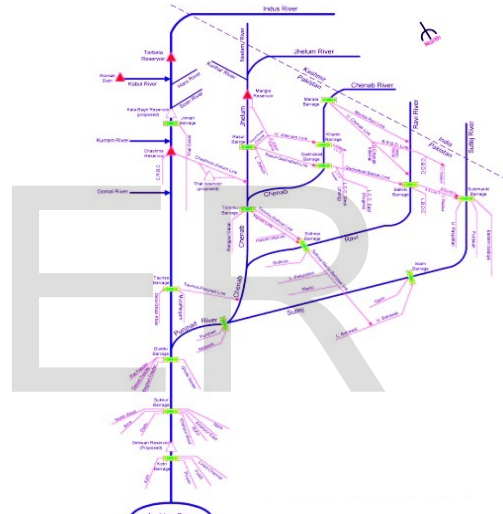


Figure 1 Irrigation Network of Pakistan [2]

Table – 1 Irrigation canals in Pakistan [3]

Province	Total length km (miles)	Design Q, cumecs (cusecs)	Command area, million ha (million miles ²)
Punjab	36,481 (22,668)	4.288 (151.36)	8.321 (0.032)
Sindh	21,192 (13,168)	3.544 (125.10)	5.101 (0.02)
NWFP	2,772 (1,722)	0.176 (6.212)	0.446 (1.8x10 ⁻³)
Baluchistan	2,655 (1,650)	0.135 (4.765)	0.384 (1.5x10 ⁻³)
Total	63,100 (39,209)	8.143 (287.44)	14.252 (0.055)

Manning's equation is the one widely used in the design for open channels. Manning's roughness coefficient 'n' is one of the most important

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parameters for describing flow resistance in open channels. Use of Manning's 'n' gives good results for fully rough and smooth conditions in rigid boundary channels but is less satisfactory in the transition range and for alluvial boundary flow as its value is highly dependant on the forms of bed roughness. Disregarding these limitations, the Manning 'n' is still probably the most commonly used in alluvial open channel flow as a resistance factor. Normally the value of 'n' is taken from tables traditionally in use, based on bed material properties.

The design of unlined channels/canals continues to be a challenge for researchers. The correct estimation of channel resistance is the main parameter which needs to be addressed, since on this depends the channel design. The error in discharge is directly proportional to the error in Manning's roughness coefficient 'n' [5]. The selection of an appropriate value for the Manning roughness factor 'n' is critical to the accuracy of channel design. For lined channels selection of a reasonably accurate value for 'n' is can be adopted but for unlined channels it is subjected to one's personal judgments and experience and in many instances the selected value may be quite inaccurate. The standard method used for the lined channels is application of Manning's equation in which the value of 'n' is determinable within reasonable limits of accuracy. However for unlined channels/canals instead of using Manning's equation directly a rational approach based on attractive force and permissible velocity methods and a statistical approach based on relationships between various parameters of flow in unlined channels are in acceptance.

Furthermore, there is a need to simplify the design procedures similar to those for lined alluvial channels where only Manning's equation leads to the desired result. This can be achieved if the flow resistance in alluvial channels can be formulated incorporating all variables related to flow, bed and sediment. In developing the flow formulae, Lacey accepted the basic Chezy formula and assuming that in alluvial channels, the resistance coefficient N was a function of the silt envelope and independent of all other factors, he developed by using Chezy and Manning's formulae, the following equation

$$V = \frac{1.346}{Na} R^{3/4} S^{1/2}$$

(1)

In which Na is a measure of the absolute resistance of the silt envelope. From 30 years data of channels in regime, Lacey calculated the value of Na from above equation and derived the equation as:

$$Na = 0.0225 f^{1/4} \quad (2)$$

Henderson [6] provides values of Manning's roughness coefficient "n" ($n=0.31d^{1/6}$), where d is characteristic particle size for unlined channels which is, however, applicable only to beds of coarse, non-cohesive material (gravel, shingles), and not applicable to fine cohesive silt of alluvial plains of major rivers like Indus, Ganges, Nile, etc. The silt factor 'f' of Lacey [4] defines the size of the sediment but not the sediment charge or the rate at which sediment is transported.

The present research is focused to develop equations which can describe in a comprehensive manner channel/canal roughness in alluvial soil including all the significant variables and parameters using a scientific and rational approach. It should be emphasized that the channel resistance for unlined alluvial channels is a complex bilateral and multilateral relationship between various channel factors like surface resistance, Reynold's number, Froude's number, channel depth and width, velocity distribution along the channel depth, channel sediment load, water level and particle size of alluvium etc. Field data collected by actual measurement for this research in 2011 [17] (March-June) from unlined canal flowing in regime. For this purpose Jamrao canal [7] in Sindh, Pakistan with discharge capacity of 102.8 m³/s (3400 ft³/sec) bottom width of 46.1 m (148 ft) and normal depth of 2.68 m (8.6 ft) which has been operating for more than hundred years was selected. Jamrao canal takes off at Jamrao Weir, located at mile 115 of Eastern Nara canal. It was constructed in 1889-1900 and started its operation in 1910 [8]. Jamrao canal is a main irrigation canal. Because of its location in the lower Indus Plains, it has a finer bed material size.

2. Objective

The main objective of this research is to develop; a “Simplified Adoptive Prediction Model for Unlined Canals (SAPMUC)”, based on rational scientific basis to offer an alternative to the existing methods in vogue, which is complex and cumbersome, also developing the prediction model for unlined canals in alluvial soil which can conveniently utilize methodologies pertaining to lined canals.

3. Literature Review:

In [9], flow resistance has been calculated for Jamrao Canal but using previous 1978 data. In this paragraph, different approaches for determining the Manning coefficient ‘n’ will be discussed but all these approaches are only for natural channel and not for man made canal in alluvium.

- “In this paper [10], Manning roughness coefficient for bare and vegetated Furrow Irrigation was calculated. This calculation was based on the volume balance equation in the form of a differential equation that was solved with the forward finite difference and secondary backward finite difference procedures”.
- “This paper [11] discussed the estimation of Manning’s roughness coefficient for basin and border irrigation from irrigation field data. The calculation was based on volume balance analysis wherein a partially differential form of the continuity equation, and Manning’s equation for open channel flow were used as governing equations. The forward difference approximation of the finite difference method was used as the solution technique”.
- “In another paper [12], a method for calculating roughness coefficient was presented, based on an analytical model for border irrigation and solved by iteration. The results were testified with the field data obtained from different regions and various soil types and situation of field surface”.
- “This paper [13] gives an overview of the meaning of the term “roughness” in the field of fluvial hydraulics and how it is often

formulated as a “resistance to flow” term in 1D, 2D and 3D numerical models. It shows how roughness is traditionally characterized in both experimental and numerical fields and subsequently challenges the definitions that currently exist”.

- “As described in [14], Air borne remote sensing has potential to provide new methods for estimating Manning’s coefficient of roughness ‘n’.
- “As described in [15] Different approaches to determine the Manning’s ‘n’ for natural stream were examined and compared. Relations between the Manning coefficient of roughness with slope and hydraulic radius were also discussed. This paper also related roughness parameter to the relative smoothness”.
- “In this paper [16] a new velocity formula in the form of a modified Manning’s formula of flow is proposed for alluvial channels. The validity of the formula is tested with 4824 sets of flume and field data covering the entire flow regime”.

4. Parameters for describing resistance to flow:

Alluvial channels are characterized by three types of variables: (1) morphological variables, which primarily relate to channel geometry and shape, which also include the description of bed forms; (2) sediment variables, which deal with channel bed material and sediment load; and (3) hydraulic variables, which pertain to hydraulic resistance to flow. There are multitudinous variables that exert the greatest influence upon the coefficient of roughness in alluvial channels [7].

$$n = f \left[\frac{Q}{VL^2}, \frac{A}{L^2}, S, \frac{R}{L}, \frac{W}{D}, \frac{SP}{V^3IL}, \frac{\tau}{IV^2}, \frac{u^*}{V}, Ntu, T, G, \frac{H}{L} \right] \quad (3)$$

Two series of field measurement were made for the reach of Jamrao canal RD 248 to 253 from March to June 2011 through

10/03/2011 14:30	194.32
10/03/2011 15:00	184.26

- (i) Remote monitoring and
- (ii) Manual observation

The objective of the field measurements through remote monitoring was to obtain, discharge, velocity, water level and turbidity, while manual measurements were carried out for cross-section, width, slope, bed forms height, and length of Jamrao canal at RD 248 – 253. Note that RD is chainage for the canal length 1RD = 311.5M (1000 ft) starting with zero RD from canal head regulator. The data were obtained from March to June 2011 at two RDs of Jamrao canal to cover the complete cycle of dry and flood season. Some of the data are listed in Table 2.1 – 2.4:

Table 2.1 Width and Slope for RD 248 – 253

At RD 248		At RD 253	
Average Width (ft)	Slope	Average Width (ft)	Slope
98	0.0001589	80.2	0.00005357
99	0.0001059	80.1	0.0001607
99.12	0.0002	80.3	0.000125
100.8	0.0005357	80.2	0.0000357
101	0.0002648	80	0.0001071

Table 2.2 Bed form heights, length and gradation coefficient for RD 248 – 253

Average Bed Form Height	Average Bed Form Length	Flatness Ratio	Gradation Coefficient
H (ft)	L (ft)	H/L	G
7.26	110	0.066	4.03
5.95	120	0.0499	1.44
5.38	135	0.0398	1.65
5.88	130	0.0452	7.15

Table 2.3: Turbidity data for RD 248

Date / Time	NTU
10/03/2011 12:30	183.41
10/03/2011 13:00	188.12
10/03/2011 13:30	184.55
10/03/2011 14:00	191.29

Table 2.4: Derived parameters for RD 248 – 253

Actual Level (ft)	Level (ft)	A (ft ²)	P(ft)	Rh=A/P
16.06	10.555	1348.62	116.11	11.615
6.23	0.73	523.32	96.46	5.425
11.12	5.621	934.164	106.242	8.793
7.48	1.982	628.488	98.964	6.351
11.55	6.045	969.78	107.09	9.056

$n=1.49S^5Rh^{.667}/V$	$U^*=(gRhS)^{.5}$	$\zeta=rRhS$	$SP=\zeta V$	$F=V/(gD)^{.5}$
0.060180255	0.243781009	0.1151671	0.1844977	0.0704579
0.039988857	0.166609562	0.0537933	0.0780541	0.1024461
0.047633571	0.212106183	0.0871837	0.1465558	0.0888317
0.048495787	0.180260163	0.0629692	0.0836861	0.0856226
0.05783384	0.2152544	0.089791	0.1267849	0.0732335

Table 2.5: Dimensionless groups for RD 248 – 253

$n=1.49S^5Rh^{.667}/V$	Q/VD^2	A/D^2	Rh/D
0.060180255	5.2320149	5.232015	0.7234519
0.039988857	13.483146	13.483146	0.8708273
0.047633571	7.5532776	7.553278	0.7906478
0.048495787	11.226945	11.226945	0.8487935
0.05783384	7.275877	7.275877	0.784387
0.083675864	5.2242055	5.224205	0.7231529

W/D	U^*/V	ζ / rV^2	$SP / V^3 \ell D$	NTU
5.2320149	0.1521729	0.0231314	0.0014408	328.83
13.483146	0.114824	0.0131702	0.002114	329.05
7.5532776	0.1261786	0.0159037	0.0014301	331.72
11.226945	0.1356359	0.0183771	0.0024562	310.25
7.275877	0.1524465	0.0232146	0.0020108	326.88

5.2242055	0.2115462	0.0447031	0.0027802	340.32
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$A = \text{Area}$, $P = \text{Wetted perimeters}$
 $R_h = \text{Hydraulic radius}$ $n = \text{Manning coefficient}$
 $U^* = \text{Shear velocity}$ $\zeta = \text{Shear stress}$
 $SP = \text{Stream power}$ $F = \text{Froude number}$

4.1 Dimensionless Analysis (Field Measurement [FM] 2011)

The data used for analysis were from May-June 2011 because data for these months are common in both reaches of canal i.e. RD 248 and 253. Some values of dimensionless groups are given in table 2.5

4.2 Fitted Line Method (Field Measurement 2011)

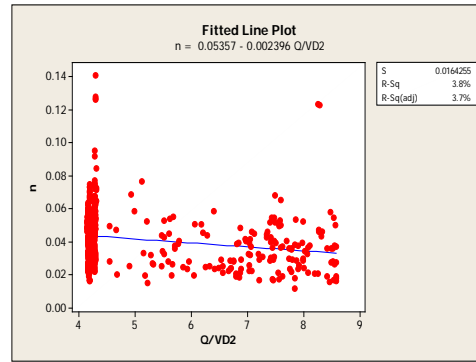
The regression equations are formed by using the field data for RD 248 -253 Jamrao canal. Some of the data are listed in Table 2.5 Minitab versions 15 is used to obtain simple regression equations between ‘n’ and dimensionless groups (Table 3).

Table 3 Regression Equations for RD 248 – 253

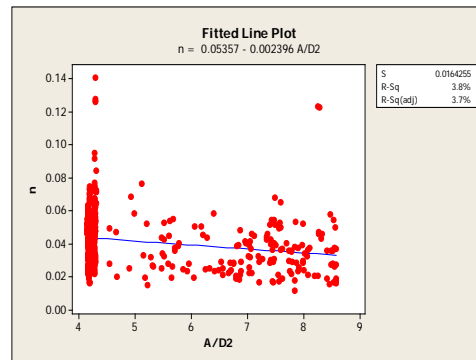
$n_1 = 0.05357 - 0.002396 Q/VD^2$
$n_2 = 0.05357 - 0.002396 A/D^2$
$n_3 = 0.02591 + 186.9 S$
$n_4 = 0.09362 - 0.07380 R/D$
$n_5 = 0.05357 - 0.002396 W/D$
$n_6 = 0.02974 + 14.37 SP/ V^3 \ell D$
$n_7 = 0.02551 + 1.25 \tau / \ell V^2$
$n_8 = 0.00173 + 0.03949 u^*/V$
$n_9 = 0.04319 - 0.000007 NTU$
$n_{10} = 0.04070 + 0.000247 T$
$n_{11} = 0.05510 - 0.002286 G$
$n_{12} = 0.04420 + 0.0726 H/L$

By putting the value of all dimensionless groups (one set of data) from Table 3 in the above equations to obtain n_1, n_2, \dots, n_{12} for the months of May & June, 2011.

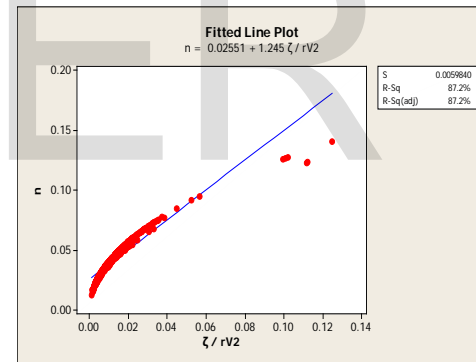
The overall final average ‘n’ for RD 248-253 for May & June is found to be 0.045 and 0.048, respectively. Selected graphs of RD 248-253 are present in Figure 2 (a-h).



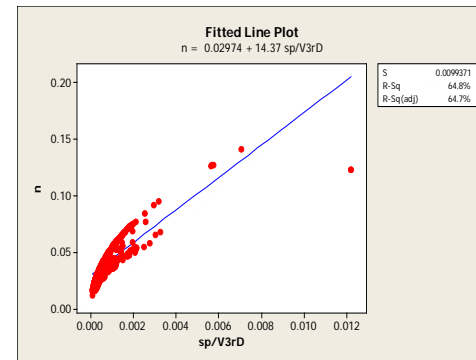
(a)



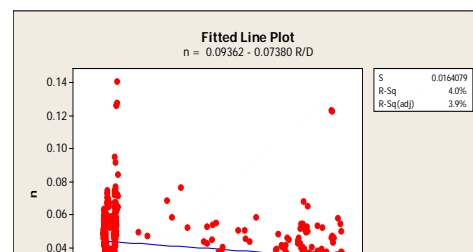
(b)



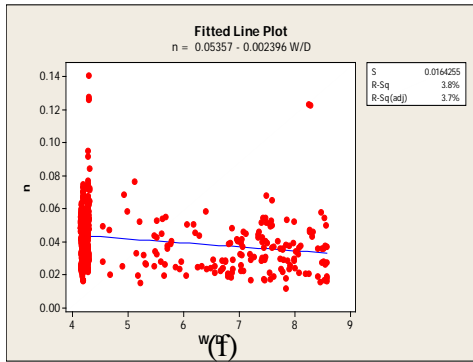
(c)



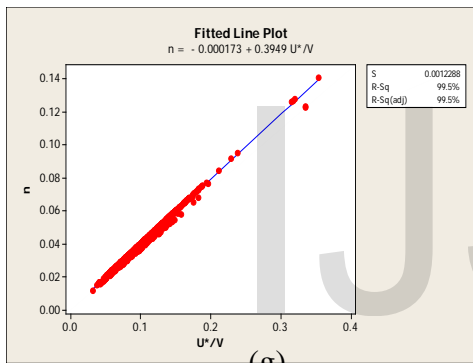
(d)



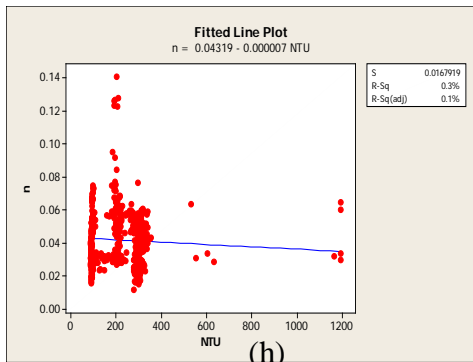
(e)



(g)



(h)



4.3 Matrix Formation Method (Field Measurement 2011)

Diagonal matrix of 12×12 was generated to obtain functional form of each dimensionless group

$$n = f \left(\begin{array}{c} 0.07055 - 0.002514 \frac{Q}{VD^2}, 0.07055 - 0.002517 \frac{A}{D^2}, 0.02591 \\ 186.9S, 0.1518 + 0.1303 \frac{R}{D}, 0.07055 - 0.002517 \frac{W}{D}, 0.03028 \\ 11.32 \frac{SP}{V^3 \ell D}, 0.02335 + 1.381 \frac{\tau}{\ell V^2}, 0.003205 + 0.4035 \frac{u}{V} \\ 0.04311 - 0.000007 \text{ NTU}, 0.04070 + 0.000247T, 0.05510 \\ 0.002286G, 0.04420 + 0.0726 \frac{H}{L} \end{array} \right) +$$

f = [A]⁻¹ [Constant]
 [A]⁻¹ = Diagonal matrix of 12×12

Functional forms of each dimensionless groups were as under.

$$\begin{array}{lll} f_1 = 28.029, & f_2 = 28.029, & f_3 = 0.000138 \\ f_4 = 1.165, & f_5 = 28.029, & f_6 = 0.00269 \\ f_7 = 0.0169, & f_8 = 0.00793, & f_9 = 6158.5 \\ f_{10} = 169.58, & f_{11} = 24.10, & f_{12} = 0.608 \end{array}$$

Using equation and Table 2.5 to obtain ‘n’ directly, ‘n’ is found to be 0.0000530 and 0.000054 for May & June respectively, if considering all dimensionless groups.

However, if those dimensionless groups (Q/VD², A/VD², R/D, W/D and S) are selected which are measured parameters not derived, and whose function values are reasonable. The value of ‘n’ is obtained as 0.036 & 0.037 for May & June, 2011, respectively.

5. Discussions:

Additionally, on the basis of recently measured field observation data of the canal from RD 248-253 in the year 2011, linear regression equations between dimensionless groups and ‘n’ were obtained for onward determination of average values of ‘n’ by ‘Fitted Line’ and ‘Matrix formation method’ as shown in Tables 4.1-3.

Table 4.1: Fitted line method

Average ‘n’	Canal reaches
0.045	RD 248-253, May 2011*
0.048	RD 248-253, June 2011*

*Observations were restricted to the month of June 2011 on account of the flooding of the canal during monsoon season

Table 4.2 Matrix formation method considering all dimensionless groups

Average ‘n’	Canal reaches
0.000053	RD 248-253, May 2011*
0.000054	RD 248-253, June 2011*

*Observations were restricted to the month of June 2011 on account of the flooding of the canal during monsoon season.

Table 4.3: Matrix formation method considering only those parameters of dimensionless groups which are measured parameters and whose functional values are in limit (Q/VD², A/D², R/D, W/D and S)

Average ‘n’	Canal reaches
0.036	RD 248-253, May 2011*
0.037	RD 248-253, June 2011*

*Observations were restricted to the month of June 2011 on account of the flooding of the canal during monsoon season.

The following reasonable functional values of the dimensionless group’s parameters were therefore selected for determining the average value of ‘n’.

For RD 248-253
 $f_1 = 28.029 \quad f_2 = 28.029 \quad f_3 = .000138$
 $f_4 = 1.165 \quad f_5 = 28.029$

The values obtained by matrix method as compared to fitted line method for FM (2011) data are consistently unrealistically low, indicating that former method is not suitable and hence not applicable to evaluate ‘n’. The main reasons appear to be that functional values of some parameters are either very high or very low, because of widely scattered data. This method shows reasonable results, if measured and reasonable functional values parameters are selected; however this attempt could not fulfill our one of the objectives

to consider maximum number of variables involved in flow to quantify the flow resistance.

The sediment load in Pakistan’s canal system is quite high in flood season (June-August) and quite low in dry weather flow condition from (October-May). These flow resistances would vary according to the sediment load. Therefore, for the purpose of canal design, an average value over a complete cycle shall be required. Allowing for the scope of this research, parameters were recorded from March to June for the year 2011. Observations were restricted up to the month of June 2011 on account of the flooding of the canal during monsoon season.

Fitted line value of ‘n’ obtained for FM data were used in the Manning’s equation. Comparison for the months of May and June given below shows that the discharge values obtained based on the calculated value of ‘n’ (Manning’s equation) are close to the observed values (Table 5).

Table 5
Comparison for the months of May & June

May		June	
By fitted line & manning equation	Observed Data	By fitted line & manning equation	Observed Data
FM (2011)	FM (2011)	FM (2011)	FM (2011)
Q(cfs)	Q(cfs)	Q(cfs)	Q(cfs)
1680	1746	2033	1891

This justifies our hypothesis that to develop a reliable predictive model, all the canal/flow variables should be taken along with their proper weightages in the form of dimensionless groups.

Table 6 presents the values of ‘n’ (which manifests flow resistance) based on FM (2011) data, values calculated by fitted line method are given only. The second of Table 6 presents the ‘n’ values as per Lacey’s equations which are in vogue presently.

Table 6
Flow resistance quantification results at a glance

By fitted line method values of	By Lacey’s method value of
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‘n’	‘n’
FM (2011)	FM (2011)
0.0465	0.0135

It is noted that Lacey’s coefficient values are consistently lower than the values obtained in this study. The possible reasons for these differences between the Lacey’s theory (and methods developed on similar lines) are ignoring important variables like bed forms and sediment load. Another potential reason may be Lacey’s non-validity for the higher range of discharge. The dimensional analysis method of estimation is independent of the range of discharge. It quantifies the flow resistance which is occurring in nature and; therefore, can prove to be more reliable in the design prediction. However, in this research the flow resistance has been quantified separately for flood season and dry weather using dimensional analysis. The use of average value of flow resistance over a period of one complete cycle for design seems to be a sound proposition. A correct value of flow resistance in the design of canal profile would cause the canal to maintain the designed shape and cross-section.

6. Conclusions:

The conclusions drawn from the study are:

- The dimensional analysis was successfully used to develop simplified adoptive prediction model (SAPMUC) for unlined alluvial channel. Fitted line method proved to be convenient mathematical tool and was successfully used to obtained final result. The alternative mathematical tool of matrix formation was found unsuitable since its produce very low values.
- SAPMUC model calculates the coefficient of resistance or ‘n’ for unlined alluvial channels thus allowing use of Manning’s equation for the design of unlined stable channels in the same way as it is used for lined sections.

- The values of 'n' obtained by SAPMUC for the Jamrao canal is 0.0456(2011), while by Lacey's method it is 0.0135.
- Typical calculated discharge values by SAPMUC for 2011 data is 1680 against the actual measured discharge of 1746. This indicates the validity of SAPMUC developed in this research.

Recommendations

- The roughness coefficient can be related to silt grade and charge; therefore, equations can be developed by using this methodology over a larger data set. This research is a step in right direction and its expansion is recommended to derive more reasonable relationships for wider application to design stable alluvial canals.
- For scattered data use of non-linear regression analysis will further improve the accuracy for calculating the coefficient of roughness.

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